**0**

**10 .... 0(2)=2^n**

**n**

**1... .... 1(2)= 10.....0(2) -1(2)= 2^n -1**

**n n**

**Integer numbers – codes and operations in complementary code**

**Example 1**

**n=8 bits**

**X= 47 = 101111(2)**

**47/2=23 23/2=11 11/2=5 5/2=2 2/2=1 1**

**47%2=1 23%2=1 11%2=1 5%2=1 2%2=0**

**Y= 96 = 64+32 = 0110 0000(2)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7 6 5 4 3 2 1 0** | | | | | | | |
| **[47] dir = [47]inv= [47]compl =** | **0** | **0** | **1** | **0** | **1** | **1** | **1** | **1** |
| **[-47]dir =** | **1** | **0** | **1** | **0** | **1** | **1** | **1** | **1** |
| **[-47]inv =** | **1** | **1** | **0** | **1** | **0** | **0** | **0** | **0** |
| **[-47]compl =** | **1** | **1** | **0** | **1** | **0** | **0** | **0** | **1** |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 6 5 4 3 2 1 0** | | | | | | | |
| **[96] dir = [96]inv= [96]compl=** | **0** | **1** | **1** | **0** | **0** | **0** | **0** | **0** |
| **[-96]dir =** | **1** | **1** | **1** | **0** | **0** | **0** | **0** | **0** |
| **[-96]inv =** | **1** | **0** | **0** | **1** | **1** | **1** | **1** | **1** |
| **[-96]compl =** | **1** | **0** | **1** | **0** | **0** | **0** | **0** | **0** |

**[47+96]compl = [47]compl  [96]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **OVERFLOW** |
| **[47]compl =** |  | **0** | **0** | **1** | **0** | **1** | **1** | **1** | **1** | **** |
| **[96]compl =** |  | **0** | **1** | **1** | **0** | **0** | **0** | **0** | **0** |  |
|  |  | **1** | **0** | **0** | **0** | **1** | **1** | **1** | **1** |  |

**[96-47]compl = [96]compl  [-47]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **2^0 + 2^4 + 2^5 = 1+16+32=49** |
| **[96]compl =** |  | **0** | **1** | **1** | **0** | **0** | **0** | **0** | **0** | **** |
| **[-47]compl =** |  | **1** | **1** | **0** | **1** | **0** | **0** | **0** | **1** |  |
|  | **~~1~~** | **0** | **0** | **1** | **1** | **0** | **0** | **0** | **1** |  |

**[47-96]compl = [47]compl  [-96]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **-(2^0 + 2^4 + 2^5) = -(1 + 16 + 32) = -49** |
| **[47]compl** |  | **0** | **0** | **1** | **0** | **1** | **1** | **1** | **1** |  |
| **[-96]compl =** |  | **1** | **0** | **1** | **0** | **0** | **0** | **0** | **0** |  |
| **[-49]compl** |  | **1** | **1** | **0** | **0** | **1** | **1** | **1** | **1** |  |
|  |  | **0** | **0** | **1** | **1** | **0** | **0** | **0** | **1** |  | **complement** |

**Subunitary numbers – codes and operations in complementary code**

**Example 2**

**n=8 bits**

**X= 0,31 = 0,0100111(2)**

**0,31 \* 2 = 0,62**

**0,62 \* 2 = 1,24**

**0,24 \* 2 = 0,48**

**0,48 \* 2 = 0,96**

**0,96 \* 2 = 1,92**

**0,92 \* 2 = 1,84**

**0,84 \* 2 = 1,68**

**Y= 0,73 = 0,1011101(2)**

**0,73 \* 2 = 1,46**

**0,46 \* 2 = 0,92**

**0,92 \* 2 = 1,84**

**0,84 \* 2 = 1,68**

**0,68 \* 2 = 1,36**

**0,36 \* 2 = 0,72**

**0,72 \* 2 = 1,44**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7, 6 5 4 3 2 1 0** | | | | | | | |
| **[0,31] dir = [0,31]inv= [0,31]compl =** | **0** | **0** | **1** | **0** | **0** | **1** | **1** | **1** |
| **[-0,31]dir =** | **1** | **0** | **1** | **0** | **0** | **1** | **1** | **1** |
| **[-0,31]inv =** | **1** | **1** | **0** | **1** | **1** | **0** | **0** | **0** |
| **[-0,31]compl =** | **1** | **1** | **0** | **1** | **1** | **0** | **0** | **1** |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7, 6 5 4 3 2 1 0** | | | | | | | |
| **[0,73] dir = [0,73]inv= [0,73]compl=** | **0** | **1** | **0** | **1** | **1** | **1** | **0** | **1** |
| **[-0,73]dir =** | **1** | **1** | **0** | **1** | **1** | **1** | **0** | **1** |
| **[-0,73]inv =** | **1** | **0** | **1** | **0** | **0** | **0** | **1** | **0** |
| **[-0,73]compl =** | **1** | **0** | **1** | **0** | **0** | **0** | **1** | **1** |

**[0,31+0,73]compl = [0,31]compl  [0,73]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S ,** | | | | | | | |  |  |
| **[0,31]compl =** |  | **0** | **0** | **1** | **0** | **0** | **1** | **1** | **1** | **** | **OVERFLOW, because the numbers are positive and the result is negative** |
| **[0,73]compl =** |  | **0** | **1** | **0** | **1** | **1** | **1** | **0** | **1** |  |
|  |  | **1** | **0** | **0** | **0** | **0** | **1** | **0** | **0** |  |

**[0,31-0,31]compl = [0,31]compl  [-0,31]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S ,** | | | | | | | |  |  |
| **[0,31]compl =** |  | **0** | **0** | **1** | **0** | **0** | **1** | **1** | **1** | **** | **Correct result: 0** |
| **[-0,31]compl =** |  | **1** | **1** | **0** | **1** | **1** | **0** | **0** | **1** |  |
|  | **~~1~~** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |  |

**[0,73-0,31]compl = [0,73]compl  [-0,31]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S ,** | | | | | | | |  |  |
| **[0,73]compl** |  | **0** | **1** | **0** | **1** | **1** | **1** | **0** | **1** | **** | **2^(-2)+2^(-3)+2^(-5) + 2^(-6) =**  **0,25+0,125+0,03125+0,015625=0,421875** |
| **[-0,31]compl =** |  | **1** | **1** | **0** | **1** | **1** | **0** | **0** | **1** |  |
|  | **1** | **0** | **0** | **1** | **1** | **0** | **1** | **1** | **0** |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

**Example 3: Represent in fixed-point notation, on 32 bits, I=14, the number 3456,73**

**3456,73 = 110 110 000 000, 101 110 101 110 000 101 000 (2)**

**3456/2 = 1728, 3456%2 = 0**

**1728/2 = 864, 1728%2 = 0**

**864/2 = 432, 864%2 = 0**

**432/2=216, 432%2=0**

**216/2=108, 216%2=0**

**108/2=54, 108%2=0**

**54/2=27, 54%2=0**

**27/2=13, 27%2=1**

**13/2=6, 13%2=1**

**6/2=3, 6%2=0**

**3/2=1, 3%2=1**

**1 / 2 = 0, 1%2 = 1**

**0,73 = 0, 5656050(8) = 0, 101 110 101 110 000 101 000**

**0,73\*8 = 5,84**

**0,84\*8=6.72**

**0.72\*8=5.76**

**0.76\*8=6.08**

**0,08\*8=0.64**

**0.64\*8=5,12**

**0,12\*8=0,96**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | I = 14 , F=17 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | | | | | B | | | | 0 | | | | 1 | | | | 7 | | | | 5 | | | | C | | | | 2 | | | |

**Example 4: Represent in floating point notation, single precision (SP), the number: 3456,73**

**3456,73 = 110 110 000 000, 101 110 101 110 000 101 (2) =**

**=0, 110 110 000 000 101 110 101 110 000 101(2) \* 2 ^ 12**

**]c = 127 + 12 = 139 =10001011 (2)**

**139/2 = 69, 139%2 =1**

**69/2 = 34, 69%2=1**

**34/2=17, 34%2=0**

**17/2=8, 17%2=1**

**8/2=4, 8%2=0**

**4/2=2, 4%2=0**

**2/2=1, 2%2=0**

**½=0, 1%2=1**

**Mantissa<1**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | c = e + 127 (8 bits) , mantisa( 23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 4 | | | | | 5 | | | | E | | | | C | | | | 0 | | | | 5 | | | | D | | | | 7 | | | |

3456,73 = **1,10 110 000 000 101 110 101 110 000 101(2) \* 2 ^ 11**

**]c = 127 + 11 = 138**

**138 = 10001010(2)**

**Mantissa>1**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | c = e + 127 (8 bits) , mantissa(23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 4 | | | | | 5 | | | | 5 | | | | 8 | | | | 0 | | | | B | | | | A | | | | E | | | |

**Example 5: Find the real number X having C504A800 its fixed-point representation on 32 bits with I=17 bits.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | I=17 bits, integer part , F=14 bits, fractional part | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | | | | 5 | | | | 0 | | | | 4 | | | | A | | | | 8 | | | | 0 | | | | 0 | | | |

X = -10001010000010010 , 10100000000000 (2) = -(2^16 +2^12+2^10+2^4+2^1 + 2^(-1) + 2^(-3))

= - (65536 + 4096+1024+16+2+½+1/8) = - (70674 +½+1/8) = - 70674,625

**Example 6:** **Find the real number X having C504A800 as its floating-point representation, SP, m>1.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | c=e+127 (8bits) , mantissa (23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | | | | | 5 | | | | 0 | | | | 4 | | | | A | | | | 8 | | | | 0 | | | | 0 | | | |

c = 10001010 (2) = 2+8+128=138

e=138-127=11

X = -1,000010010101 (2) \* 2 ^ 11 = -100001001010,1(2) = -(2+8+64+2048+0.5)= -2122,5